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Procedia Engineering 15 (2011) 2836 – 2840

Procedia Engineering

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Advanced in Control Engineeringand Information Science

Stability Criterion for BAM Neural Networks of Neutral-Type with Interval Time-Varying Delays

Guoquan Liu[a*], Simon X. Yang[a,b]

[a]*College of Automation,Chongqing University, Chongqing 400044,China*
[b]*School of Engineering ,University of Guelph, Guelph, Ontario, Canada*

Abstract

In this paper, the asymptotic stability for bidirectional associative memory (BAM) neural networks of neutral-type with interval time-varying delays is investigated. The discrete delay is assumed to be time-varying and belong to a given interval, which means that the lower and upper bounds of interval time-varying delays are available. By employing the Lyapunov-Krasovskii functional method and using the linear matrix inequality (LMI) technique, a new delay-range-dependent stability criterion is established in terms of LMI. In addition, the proposed LMI based results can be easily checked by LMI control toolbox in Matlab.

Selection and/or peer-review under responsibility of [CEIS 2011]

*Keywords:*Asymptotic stability; Bidirectional associative memory neural networks; Neutral-type; Linear matrix inequality; Interval time-varying delays;

1. Introduction

It is well known that bidirectional associative memory (BAM) neural network is a type of recurrent neural network. BAM neural network was introduced by [1]-[2]. During the past years, the dynamics of BAM neural networks have been widely studied due to their extensive applications in many areas such as associative memory, pattern recognition, optimization and automatic control. In practice, time delays are likely to be present due to the finite switching speed of amplifiers and occur in signal transmission among

* Corresponding author. Address: College of Automation, Chongqing University, Chongqing 400044, China.
E-mail address: guoquanliu1982@hotmail.com.

doi:10.1016/j.proeng.2011.08.534

neurons in the electronic implementation of neural networks, In addition, time delay is often a source of oscillations, chaos and instability in various types of neural networks. Thus, the study of the stability problem of BAM neural networks with time delays has received great attention in recent years and a number of results have been reported [3]-[7].

On the other hand, many dynamical neural networks are described with neutral functional differential equations that include neutral delay differential equations. These neural networks are called neutral neural networks or neural networks of neural-type. Recently, a few results about the global asymptotic or exponential stability for BAM neural networks of neural-type have been derived in the literatures [8]-[10]. In [8], a delay-dependent global asymptotic stability criterion is presented for BAM neural networks of neutral-type by using the Lyapunov method. In [9], Liu and zhang furthermore investigated asymptotic stability for BAM neural networks of neutral-type, a novel delay-dependent stability conditions were established. In [10], by utilizing the Lyapunov-krasoviskii functional and combining with the LMI approach; three sufficient conditions were given ensuring the global exponential stability for BAM neural networks of neutral-type with time-varying delays. Up to now, the asymptotic stability problem has not been touched for BAM neural networks of neutral-type with interval time-varying delays, which is still open problem.

Based the aforementioned discussions, a class of BAM neural networks of neutral-type with interval time-varying delays is considered in this paper. Based on the Lyapunov-krasoviskii stability method and the LMI technique, a new asymptotic stability criterion is presented in terms of LMI.

Notations: The notations are quite standard. $\Re^n$ and $\Re^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively; For a real symmetric matrix X , the notation $X\geq 0$ (respectively, $X>0$), means that X is positive semi-definite (respectively, positive definite); The superscripts "T" and "-1" stand for matrix transposition and matrix inverse, respectively; The mathematical expectation operator with respect to the given probability measure P is denoted by $E\{\cdot\}$. diag$\{\cdots\}$ denotes a block diagonal matrix; $*$ denotes the elements below the main diagonal of a symmetric block matrix.

2. Problem Formulation

Consider the following BAM neural networks of neutral-type with interval time-varying delays:

$$\begin{aligned}\dot{u}(t)&=-Au(t)+W_1\overline{f}(v(t))+W_2\overline{f}(v(t-\tau(t)))+W_3\dot{v}(t-h_1(t))+I,\\ \dot{v}(t)&=-Bv(t)+V_1\overline{g}(u(t))+V_2\overline{g}(u(t-\sigma(t)))+V_3\dot{u}(t-h_2(t))+J,\end{aligned}\tag{1}$$

where $u=[u_1,u_2,...,u_n]^T$ and $v=[v_1,v_2,...,v_n]^T$ are the neuron state vectors. $A=\mathrm{diag}\{a_1,a_2,...,a_n\}>0,$ $B=\mathrm{diag}\{b_1,b_2,...,b_m\}>0,$ W_1,W_2,W_3,V_1,V_2 and V_3 are known constant matrices with appropriate dimension, $\overline{f}$ and $\overline{g}$ denote the neuron activations, I and J denote the constant external inputs. $\tau(t)$ and $\sigma(t)$ represent the discrete transmission delays with

$$0\leq\tau_1\leq\tau(t)\leq\tau_2,\dot{\tau}(t)\leq\tau_d<1,0\leq\sigma_1\leq\sigma(t)\leq\sigma_2,\dot{\tau}(t)\leq\sigma_d<1,\tag{2}$$

where $\tau_1,\tau_2,\tau_d,\sigma_1,\sigma_2$ and σ_d are constants.

$h_1(t)$ and $h_2(t)$ represent the neutral delays with

$$0\leq h_1(t)\leq h_1,\dot{h}_1(t)\leq h_{1d}<1,0\leq h_2(t)\leq h_2,\dot{h}_2(t)\leq h_{2d}<1.\tag{3}$$

Assume that the neuron activation functions $\overline{f}$ and $\overline{g}$ satisfy the following hypotheses

(A1) $\overline{f}$ and $\overline{g}$ are bounded functions.

(A2) $\overline{f}$ and $\overline{g}$ are Lipschitz continuous, i.e., there exist real scalars $l_j>0$ and $k_i>0$, such that

$\left|\overline{f}(\varsigma_1)-\overline{f}(\varsigma_2)\right| \le l_j|\varsigma_1-\varsigma_2|$, $|\overline{g}(\varsigma_1)-\overline{g}(\varsigma_2)| \le k_i|\varsigma_1-\varsigma_2|$, for all $\varsigma_1,\varsigma_2 \in \Re$ and $\varsigma_1 \neq_2$ for any $i=1,2,...,n,\ j=1,2,...,m.$ It is clear that under the assumptions (A1) and (A2), system (1) has at least one equipment point. Suppose that $u^*=\left(u_1^*,u_2^*,...,u_m^*\right)^T$ and $v^*=\left(v_1^*,v_2^*,...,v_n^*\right)^T$ be one equilibrium point of system (1).

For convenience, we shift u^*, v^* to the origin by taken the following transformation:

$$x(\cdot)=u(\cdot)-u^*, f(\cdot)=\overline{f}(u(\cdot))-\overline{f}(u^*(\cdot)), y(\cdot)=v(\cdot)-v^*, g(v(\cdot))=\overline{g}(v(\cdot))-\overline{g}(u^*(\cdot)). \tag{4}$$

Then, system (1) can be written as

$$\begin{aligned} \dot{x}(t) &= -Ax(t)+W_1f(y(t))+W_2f(y(t-\tau(t)))+W_3\dot{y}(t-h_1(t)),\\ \dot{y}(t) &= -By(t)+V_1g(x(t))+V_2g(x(t-\tau(t)))+V_3\dot{x}(t-h_2(t)). \end{aligned} \tag{5}$$

From (A1) and (A2), we can derive that activation f and g satisfy

(G1) f and g are bounded functions.

(G2) f and g are Lipschitz continuous, i.e., there exist real scalars $l_j>0$ and $k_i>0$, such that $|f(\varsigma_1)-f(\varsigma_2)| \le l_j|\varsigma_1-\varsigma_2|$, $|g(\varsigma_1)-g(\varsigma_2)| \le k_i|\varsigma_1-\varsigma_2|$, for all $\varsigma_1,\varsigma_2 \in \Re$ and $\varsigma_1 \neq \varsigma_2$ for any $i=1,2,...,n,\ j=1,2,...,m$ with $f(0))=0, g(0))=0.$

In order to obtain the main result, a basic lemma is always made throughout this paper.

Lemma 1. For any constant matrix $M>0$, any scalars a and b with $a<b$, and a vector function $x(t):[a,b]\to\Re^n$ such that the integrals concerned as well defined, the following holds:

$$\left[\int_a^b x(s)ds\right]^T M\left[\int_a^b x(s)ds\right] \le (b-a)\int_a^b x^T(s)Mx(s)ds. \tag{6}$$

3. Stability Analysis

In this section, we propose a new stability criterion for BAM neural networks of neutral-type with interval time-varying delays described in (5).

Theorem 1. Under assumptions (G1) and (G2) hold. The equilibrium point of system (5) is asymptotically stable, if there exist positive definite matrices $P_1,P_2,\ Q_i, i=1,2,...,6$, $R_i, i=1,2,3,4, Z_1,Z_2,$ and diagonal positive definite matrices M_1,M_2 such that the following LMI holds:

$$\Omega=\begin{bmatrix} \Omega_1 & 0 & \Omega_2 & 0\\ * & -\Omega_3 & 0 & 0\\ * & * & \Omega_4 & 0\\ * & * & * & -\Omega_5 \end{bmatrix}<0, \tag{7}$$

where

$$\Omega_1=\begin{bmatrix} \Xi_1 & 0 & \Xi_2 & \Xi_3\\ * & \Xi_4 & \Xi_5 & \Xi_6\\ * & * & \Xi_7 & \Xi_8\\ * & * & * & \Xi_9 \end{bmatrix}, \Omega_2=\begin{bmatrix} \Xi_{10} & 0 & \Xi_{11} & 0\\ 0 & \Xi_{12} & 0 & \Xi_{13}\\ \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17}\\ \Xi_{18} & \Xi_{19} & \Xi_{20} & \Xi_{21} \end{bmatrix}, \Omega_3=\mathrm{diag}\{\Xi_{22}\quad \Xi_{23}\quad R_1\quad R_2\quad R_3\quad R_4\},$$

$$\Omega_4=\begin{bmatrix} \Xi_{24} & 0 & \Xi_{25} & 0\\ * & \Xi_{26} & 0 & \Xi_{27}\\ * & * & \Xi_{28} & 0\\ * & * & * & \Xi_{29} \end{bmatrix}, \Omega_5=\mathrm{diag}\{Z_1\quad Z_2\},$$

with

$\Xi_1 = -P_1A - AP_1^T + Q_1 + A^TQ_6A + R_1 + R_2 + (\tau_2-\tau_1)^2 Z_1, \Xi_2 = P_1W_1 - A^TQ_6W_1, \Xi_3 = -AM_2^T,$
$\Xi_4 = P_1W_2 - A^TQ_6W_2, \Xi_5 = P_1W_3 - A^TQ_6W_3, \Xi_6 = Q_3 - P_2B - BP_2^T + B^TQ_5B + R_3 + R_4 + (\sigma_2-\sigma_1)^2 Z_2,$
$\Xi_7 = -BM_1^T, \Xi_8 = P_1V_1 - BM_2^T - B^TQ_5V_1, \Xi_9 = P_2V_2 - B^TQ_5V_2, \Xi_{10} = P_2V_3 - B^TQ_5V_3,$
$\Xi_{11} = Q_2 + M_1W_1 + W_1^TM_1^T + W_1^TQ_6W_1, \Xi_{12} = M_1V_1 + W_1^TM_2^T, \Xi_{13} = W_1^TQ_6W_2, \Xi_{14} = M_1V_2, \Xi_{15} = W_1^TQ_6W_3,$
$\Xi_{16} = M_1V_3 + W_1^TQ_6W_3, \Xi_{17} = Q_4 + M_2V_1 + V_1^TM_2^T + V_1^TQ_5V_1, \Xi_{18} = M_2W_2 + V_1^TQ_5V_2, \Xi_{19} = V_1^TQ_5V_2,$
$\Xi_{20} = M_2W_3 + V_1^TQ_5V_3, \Xi_{21} = V_1^TQ_5V_3, \Xi_{22} = (1-\tau_d)Q_1, \Xi_{23} = (1-\sigma_d)Q_3, \Xi_{24} = -(1-\tau_d)Q_2 + W_2^TQ_6W_2,$
$\Xi_{25} = W_2^TQ_6W_3, \Xi_{26} = -(1-\sigma_d)Q_4 + V_2^TQ_6V_2, \Xi_{27} = V_2^TQ_5V_3, \Xi_{28} = -(1-h_{1d})Q_5 + W_3^TQ_6W_3, \Xi_{29} = -(1-h_{2d})Q_6 + V_3^TQ_5V_3.$

Proof. Construct a lyapunov-krasoskill functional for system (5) as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{8}$$

where

$$V_1(t) = x^T(t)P_1x(t) + y^T(t)P_2y(t) + 2\sum_{j=1}^{n} m_{1j}\int_0^{y_j(t)} f_j(s)ds + 2\sum_{i=1}^{n} m_{2i}\int_0^{x_i(t)} g_i(s)ds,$$

$$V_2(t) = \int_{t-\tau(t)}^{t}\left[x^T(s)Q_1x(s) + f^T(y(s))Q_2f(y(s))\right]ds + \int_{t-\sigma(t)}^{t}\left[y^T(s)Q_3y(s) + g^T(x(s))Q_4g(x(s))\right]ds$$
$$+ \int_{t-h_1(t)}^{t}\dot{y}^T(s)Q_5\dot{y}(s)ds + \int_{t-h_2(t)}^{t}\dot{x}^T(s)Q_6\dot{x}(s)ds,$$

$$V_3(t) = \int_{t-\tau_1}^{t}x^T(s)R_1x(s)ds + \int_{t-\tau_2}^{t}x^T(s)R_2x(s)ds + \int_{t-\sigma_1}^{t}y^T(s)R_3y(s)ds + \int_{t-\sigma_2}^{t}y^T(s)R_4y(s)ds,$$

$$V_4(t) = (\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+\delta}^{t}x^T(s)Z_1x(s)dsd\delta + (\sigma_2-\sigma_1)\int_{-\sigma_2}^{-\sigma_1}\int_{t+\delta}^{t}y^T(s)Z_2y(s)dsd\delta.$$

Calculating the derivative of $V(t)$ along the trajectory of system (6) is

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t), \tag{9}$$

where

$$\begin{aligned}\dot{V}_1(t) &= 2x^T(t)P_1\left[-Ax(t) + W_1f(y(t)) + W_2f(y(t-\tau(t))) + W_3\dot{y}(t-h_1(t))\right]\\ &+ 2y^T(t)P_2\left[-By(t) + V_1g(x(t)) + V_2g(x(t-\sigma(t))) + V_3\dot{x}(t-h_2(t))\right] + 2M_1f^T(y(t))\\ &\times\left[-By(t) + V_1g(x(t)) + V_2g(x(t-\tau(t))) + V_3\dot{x}(t-h_2(t))\right] + 2M_2g^T(x(t))\left[-Ax(t)\right.\\ &\left.+W_1f(y(t)) + W_2f(y(t-\tau(t))) + W_3\dot{y}(t-h_1(t))\right],\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{V}_2(t) &= x^T(t)Q_1x(t) - (1-\dot{\tau}(t))x^T(t-\tau(t))Q_1x(t-\tau(t)) + f^T(y(t))Q_2f(y(t)) - (1-\dot{\tau}(t))f^T(y(t-\tau(t)))\\ &\times Q_2f^T(y(t-\tau(t))) + y^T(t)Q_3y(t) - (1-\dot{\sigma}(t))y^T(t-\tau(t))Q_3y(t-\tau(t)) + g^T(x(t))Q_4g(x(t))\\ &- (1-\dot{\sigma}(t))g^T(x(t-\sigma(t)))Q_4g(x(t-\sigma(t))) + \left[-By(t) + V_1g(x(t)) + V_2g(x(t-\tau(t)))\right.\\ &\left.+V_3\dot{x}(t-h_2(t))\right]^TQ_5\left[-By(t) + V_1g(x(t)) + V_2g(x(t-\tau(t))) + V_3\dot{x}(t-h_2(t))\right]\\ &- (1-\dot{h}_1(t))\dot{y}^T(t-h_1(t))Q_5\dot{y}(t-h_1(t)) + \left[-Ax(t) + W_1f(y(t)) + W_2f(y(t-\tau(t)))\right.\\ &\left.+W_3\dot{y}(t-h_1(t))\right]^TQ_6\left[-Ax(t) + W_1f(y(t)) + W_2f(y(t-\tau(t))) + W_3\dot{y}(t-h_1(t))\right]\\ &- (1-\dot{h}_2(t))\dot{x}^T(t-h_2(t))Q_6\dot{x}(t-h_2(t)),\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_3(t) &= x^T(t)R_1x(t) - x^T(t-\tau_1)R_1x(t-\tau_1) + x^T(t)R_2x(t) - x^T(t-\tau_2)R_2x(t-\tau_2) + y^T(t)R_3y(t)\\ &- y^T(t-\sigma_1)R_3y(t-\sigma_1) + y^T(t)R_4y(t) - y^T(t-\sigma_2)R_4y(t-\sigma_2),\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{V}_4(t) &= (\tau_2-\tau_1)^2x^T(t)Z_1x(t) - (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}x^T(s)Z_1x(s)ds + (\sigma_2-\sigma_1)^2y^T(t)Z_2y(t)\\ &- (\sigma_2-\sigma_1)\int_{t-\sigma_2}^{t-\sigma_1}y^T(s)Z_2y(s)ds.\end{aligned} \tag{13}$$

By Lemma 1, we known that

$$-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}x^T(s)Z_1x(s)ds \le -\left[\int_{t-\tau_2}^{t-\tau_1}x(s)ds\right]^T Z_1\left[\int_{t-\tau_2}^{t-\tau_1}x(s)ds\right], \tag{14}$$

$$-(\sigma_2-\sigma_1)\int_{t-\sigma_2}^{t-\sigma_1} y^T(s)Z_2 y(s)ds \le -\left[\int_{t-\sigma_2}^{t-\sigma_1} y(s)ds\right]^T Z_2\left[\int_{t-\sigma_2}^{t-\sigma_1} y(s)ds\right]. \qquad (15)$$

By utilizing relationships (9)-(15), we have

$$dV(t) \le \xi^T(t)\Omega\xi(t)dt, \qquad (16)$$

where Ω is defined in (9)
and

$$\xi^T(t)=\Big[x^T(t), y^T(t), f^T(y(t)), g^T(x(t)), x^T(t-\tau(t)), y^T(t-\tau(t)), x^T(t-\tau_1), x^T(t-\tau_2), y^T(t-\sigma_1),$$
$$y^T(t-\sigma_2), f^T(y(t-\tau(t))), g^T(x(t-\sigma(t))), \dot{y}(t-h_1(t)), \dot{x}(t-h_2(t)), \left(\int_{t-\tau_2}^{t-\tau_1} x(s)ds\right)^T, \left(\int_{t-\sigma_2}^{t-\sigma_1} y(s)ds\right)^T\Big].$$

It is obvious that for $\Omega<0$, which indicates from the Lyapunov stability theory that the system (5) is asymptotic stable. This completes the proof.

4. Conclusion

This paper addresses the problem of the asymptotic stability for BAM neural networks of neutral-type with interval time-varying delays. A new delay-range-dependent asymptotic stability condition for the considered systems is proposed in terms of LMI based on the Lyapunov-Krasovskii function method and the LMI technique.

Acknowledgment. This work was supported by the Fundamental Research Funds for the Central Universities (No. CDJXS11172237).

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